Collective Effects on the Transition Bremsstrahlung Spectrum due to the Polarization Interaction in Nonideal Plasmas

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Z. Naturforsch. **64a,** 49 – 53 (2009); received February 22, 2008 / revised June 30, 2008

The collective effects on the transition bremsstrahlung spectrum due to the polarization interaction between the electron and Debye shielding cloud of an ion are investigated in nonideal plasmas. The impact parameter analysis with the effective pseudopotential model taking into account the nonideal collective and plasma screening effects is applied to obtain the bremsstrahlung radiation cross-section as a function of the nonideality plasma parameter, Debye length, photon energy, and projectile energy. It is shown that the collective effects enhance the bremsstrahlung radiation cross-section and decrease with increasing impact parameter. It is also shown that the collective effect is the most significant near the maximum position of the bremsstrahlung cross-section. In addition, it is shown that the collective effect decreases with an increase of the radiation photon energy.

Key words: Transition Bremsstrahlung; Nonideal Plasmas.

The bremsstrahlung process [1 – 3] has been of great interest since this process is one of the most fundamental processes in many areas of physics such as astrophysics, atomic physics, and plasma physics. In addition, the inverse process to the bremsstrahlung has been also of interest, especially in astrophysical plasmas [1]. The transition bremsstrahlung or so-called polarization bremsstrahlung [4 – 6] in plasmas, due to the interaction between the Debye shielding cloud and plasma particle, has been extensively investigated in weakly and strongly coupled plasmas since this process has provided useful information on various plasma parameters. For low-energy electron encounters, the transition bremsstrahlung is expected to be significant due to the long collision time between the projectile electron and induced polarization charge in the Debye shielding cloud. The plasma described by the Debye-Hückel model has been classified as the ideal plasma since the average interaction energy between charged particles is quite smaller than the average kinetic energy of a particle [7]. Moreover, the multiparticle correlation effects caused by simultaneous interactions of charged particles should be taken into account with increasing the plasma density. In plasmas with moderate densities and temperatures, the potential energy would not be well characterized by the conventional Debye-Hückel model due to the strong collective effects of nonideal particle interactions [8 – 11]. Hence, it is expected that the transition bremsstrahlung processes in nonideal plasmas would be quite different from those in ideal plasmas. Thus, we now will investigate the nonideal collective effects on the transition bremsstrahlung due to the polarization interaction between the projectile electron and Debye shielding cloud of an ion in a nonideal plasma using the effective pseudopotential model and taking into account the collective and plasma screening effects since theoretical atomic spectroscopy is crucial to the study of various plasma parameters. The impact parameter trajectory analysis is applied in order to obtain the bremsstrahlung radiation cross-section as a function of the nonideality plasma parameter, Debye length, photon energy, and projectile energy.

In the low-energy bremsstrahlung process, the bremsstrahlung cross-section [12] $\mathrm{d}\sigma_\mathrm{b}$ can be written in the form

$$\mathrm{d}\sigma_\mathrm{b} = 2\pi \int \mathrm{d}b\, b\, \mathrm{d}w_\omega(b), \qquad (1)$$

where b is the impact parameter and $\mathrm{d}w_\omega(b)$ is the differential photon emission probability with frequency ω. For all impact parameters, the probability $\mathrm{d}w_\omega(b)$ would be obtained by the Larmor formula [13] for the instantaneous power emitted during the en-

0932–0784 / 09 / 0100–0049 $ 06.00

counter of the projectile electron with the target system:

$$\mathrm{d}w_\omega(b) = \frac{8\pi e^2}{3\hbar m^2 c^3}|\mathbf{F}_\omega(b)|^2\frac{\mathrm{d}\omega}{\omega}, \tag{2}$$

where m is the electron mass and $\mathbf{F}_\omega(b)$ is the Fourier coefficient of the force $\mathbf{F}(t)$ acting on the projectile electron:

$$\mathbf{F}_\omega(b) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t}\mathbf{F}(t). \tag{3}$$

Here, the absolute square $|\mathbf{F}_\omega(b)|^2$ can be decomposed into the Fourier coefficients perpendicular, $\mathbf{F}_{\perp\omega}$, and parallel, $\mathbf{F}_{\parallel\omega}$, to the direction of the projectile velocity $\mathbf{v}_0$:

$$\begin{aligned}&|\mathbf{F}_\omega(b)|^2 = |\mathbf{F}_{\perp\omega}|^2 + |\mathbf{F}_{\parallel\omega}|^2 = \\ &|(\mathbf{v}_0\times\mathbf{F}_\omega)\times\mathbf{v}_0/v_0^2|^2 + |\mathbf{v}_0(\mathbf{v}_0\cdot\mathbf{F}_\omega)/v_0^2|^2.\end{aligned} \tag{4}$$

The integro-differential equation [8] for the effective potential of charged particle interactions taking into account the simultaneous correlations of many particles has been obtained on the basis of a sequential solution of the Bogolyubov chain equations for the equilibrium distribution function of particles in nonideal plasmas. In addition, the remarkably useful analytical form of the pseudopotential of the particle interaction in nonideal plasmas has been also obtained by the application of the spline-approximation [8]. Using the pseudopotential model [8] and taking into account the collective and plasma screening effects, the potential $\phi(r)$ due to the test charge q_T in nonideal plasmas is given by

$$\phi(r) = \frac{q_\mathrm{T}}{r}\mathrm{e}^{-r/\Lambda}\frac{1+\gamma f(r)/2}{1+c(\gamma)}, \tag{5}$$

where r is the distance from the test charge, Λ is the Debye length, γ ($\equiv e^2/\Lambda k_\mathrm{B}T$) is the nonideality plasma parameter, k_B is the Boltzmann constant, T is the electron temperature, $f(r) = (\mathrm{e}^{-\sqrt{\gamma}r/\Lambda}-1)(1-\mathrm{e}^{-2r/\Lambda})/5$, and $c(\gamma) \cong 0.456\gamma - 0.108\gamma^2 + 0.009\gamma^3$ is the correlation coefficient. In this pseudopotential model, the ranges of the electron number density and temperature are, respectively, known to be less than 10^{19} cm^{-3} and 10^4-10^5 K, and the nonideal coupling plasma parameter becomes $\gamma \le 1$ [8]. Here, we assume that the plasma is fully ionized so that it is composed of electrons and ions. Thus, the interaction between the charged particles with neutral atoms is ignored in the present work. The effective potentials for the charge-atom interactions can be found in a recent excellent investigation [14]. In ideal or weakly nonideal plasmas, i. e., in the case of $\gamma \to 0$, the pseudopotential (5) goes over into the conventional Debye-Hückel potential $\phi(r) \to (q_r/r)\mathrm{e}^{-r/\Lambda}$. Since the electron number density $n_\mathrm{e}(r)$ inside the Debye shielding cloud which contains the ion with nuclear charge Ze and plasma electrons in nonideal plasmas can be represented by

$$n_\mathrm{e}(r) = \frac{Z}{4\pi\Lambda^2}\frac{\mathrm{e}^{-r/\Lambda}}{r}\frac{1+\gamma f(r)/2}{1+c(\gamma)}, \tag{6}$$

we obtain the polarization force $\mathbf{F}_\mathrm{pol}(\mathbf{r})$ acting on the projectile electron due to the polarized Debye shielding cloud in the following form:

$$\begin{aligned}\mathbf{F}_\mathrm{pol}(\mathbf{r}) = &-\nabla\left[-\frac{e^2}{r^2}\int_{r'\le r}\mathrm{d}^3\mathbf{r}'r'n_\mathrm{e}(r')\right] = -\frac{2Ze^2}{\Lambda^2}\frac{1}{1+c(\gamma)}\frac{\mathbf{r}}{r^4}\left\{\left[2\Lambda^3 - \left(\frac{r^3}{2}+r^2\Lambda+2r\Lambda^2+2\Lambda^3\right)\mathrm{e}^{-r/\Lambda}\right]\right.\\ &+\frac{\gamma}{10}\left[\left[\frac{2\Lambda^3}{(\sqrt{\gamma}+1)^3} - \left(\frac{r^3}{2}+\frac{r^2\Lambda}{\sqrt{\gamma}+1}+\frac{2r\Lambda^2}{(\sqrt{\gamma}+1)^2}+\frac{2\Lambda^3}{(\sqrt{\gamma}+1)^3}\right)\mathrm{e}^{-(\sqrt{\gamma}+1)r/\Lambda}\right]\right.\\ &-\left[\frac{2\Lambda^3}{(\sqrt{\gamma}+3)^3} - \left(\frac{r^3}{2}+\frac{r^2\Lambda}{\sqrt{\gamma}+3}+\frac{2r\Lambda^2}{(\sqrt{\gamma}+3)^2}+\frac{2\Lambda^3}{(\sqrt{\gamma}+3)^3}\right)\mathrm{e}^{(\sqrt{\gamma}+3)r/\Lambda}\right]\\ &\left.\left.-\left[2\Lambda^3 - \left(\frac{r^3}{2}+r^2\Lambda+2r\Lambda^2+2\Lambda^3\right)\mathrm{e}^{-r/\Lambda}\right] + \left[\frac{2\Lambda^3}{27} - \left(\frac{r^3}{2}+\frac{r^2\Lambda}{3}+\frac{2r\Lambda^2}{9}+\frac{2\Lambda^3}{27}\right)\mathrm{e}^{-3r/\Lambda}\right]\right]\right\},\end{aligned} \tag{7}$$

where $\mathbf{r} = \mathbf{b} + \mathbf{v}_0 t$. After a considerable amount of some algebra using the impact parameter analysis, we, respectively, obtain the scaled perpendicular, $\bar{F}_{\perp\omega}$, and parallel, $\bar{F}_{\parallel\omega}$, Fourier coefficients of the polarization force as

$$\bar{F}_{\perp\omega} \equiv -\frac{\pi a_Z v_0}{2Ze^2}\left(\frac{\mathbf{b}\cdot\mathbf{F}_\omega}{b}\right) = \int_0^\infty \mathrm{d}\tau \bar{b}\cos(\xi\tau) G(\bar{r},\bar{\Lambda},\gamma), \quad (8)$$

$$\bar{F}_{\parallel\omega} \equiv -\frac{\pi a_Z v_0}{2Ze^2}\left(\frac{\mathbf{v}_0\cdot\mathbf{F}_\omega}{v_0}\right) = \mathrm{i}\int_0^\infty \mathrm{d}\tau \tau \sin(\xi\tau) G(\bar{r},\bar{\Lambda},\gamma), \quad (9)$$

where τ ($\equiv v_0 t/a_Z$) is the scaled time, a_Z ($= a_0/Z$) is the first Bohr radius of the hydrogenic ion with nuclear charge Ze, a_0 ($= \hbar^2/me^2$) is the Bohr radius of the hydrogen atom, $\bar{b}$ ($\equiv b/a_Z$) is the scaled impact parameter, $\xi \equiv \omega a_Z/v_0$, $\bar{r}$ [($\equiv r/a_Z$) $= (\bar{b}^2+\tau^2)^{1/2}$] is the scaled distance, $\bar{\Lambda}$ ($\equiv \Lambda/a_Z$) is the scaled Debye length, and the integrand function $G(\bar{r},\bar{\Lambda},\gamma)$ is given by

$$G(\bar{r},\bar{\Lambda},\gamma) = \frac{1}{1+c(\gamma)}\frac{1}{\bar{\Lambda}^2\bar{r}^4}\Bigg\{\Bigg[2\bar{\Lambda}^3 - \left(\frac{\bar{r}^3}{2} + \bar{r}^2\bar{\Lambda} + 2\bar{r}\bar{\Lambda}^2 + 2\bar{\Lambda}^3\right)\mathrm{e}^{-\bar{r}/\bar{\Lambda}}\Bigg] + \frac{\gamma}{10}\sum_{p=0}^{1}\sum_{q=0}^{1}(-1)^{p+q+1} \cdot\Bigg[\frac{2\bar{\Lambda}^3}{\left(p\sqrt{\gamma}+2q+1\right)^3} - \left(\frac{\bar{r}^3}{2} + \frac{\bar{r}^2\bar{\Lambda}}{p\sqrt{\gamma}+2q+1} + \frac{2\bar{r}\bar{\Lambda}^2}{\left(p\sqrt{\gamma}+2q+1\right)^2} + \frac{2\bar{\Lambda}^3}{\left(p\sqrt{\gamma}+2q+1\right)^3}\right) \cdot\mathrm{e}^{-\left(p\sqrt{\gamma}+2q+1\right)\bar{r}/\bar{\Lambda}}\Bigg]\Bigg\}. \quad (10)$$

On substituting (2), (8), and (9) into (1), we obtain the following form of the transition bremsstrahlung cross-section:

$$\mathrm{d}\sigma_\mathrm{b} = \frac{2^6}{3}\frac{\alpha^3 a_0^2}{\bar{E}}\frac{\mathrm{d}\omega}{\omega}\int \mathrm{d}\bar{b}\,\bar{b}\left(|\bar{F}_{\perp\omega}|^2 + |\bar{F}_{\parallel\omega}|^2\right), \quad (11)$$

where α ($= e^2/\hbar c \approx 1/137$) is the fine structure constant, $\bar{E} \equiv E/Z^2Ry$, E ($= mv_0^2/2$) is the kinetic energy of the projectile electron, and Ry ($= me^4/2\hbar^2 \approx$ 13.6 eV) is the Rydberg constant. It has been known that the classical trajectory method is quite useful to investigate the low-energy bremsstrahlung process [15]. Hence, the expression of the bremsstrahlung cross-section [equation (11)] would be reliable to investigate the low-energy bremsstrahlung process due to the polarization interactions in nonideal plasmas. Very recently, an excellent discussion on the Gaunt factor and comparisons for the bremsstrahlung calculations by using various methods such as Born, Kramers, and Sommerfeld were given by Fortmann, Redmer, Reinholz, Röpke, Wierling, and Rozmus [16]. It has been shown that the bremsstrahlung emission spectrum would be investigated through the bremsstrahlung radiation cross-section [13] defined as $\mathrm{d}\chi_\mathrm{b}/\mathrm{d}\bar{\varepsilon} \equiv (\mathrm{d}\sigma_\mathrm{b}/\hbar\mathrm{d}\omega)\hbar\omega$. Hence, the bremsstrahlung radiation cross-section would be extended to the soft-photon limit due to the cancellation of the factor $\mathrm{d}\omega/\omega$ in the photon emission probability (2). After some algebra, the scaled differential transition bremsstrahlung radiation cross-section in units of πa_0^2 due to the polarization interaction between the electron and Debye shielding cloud of an ion in nonideal plasmas is found to be

$$\frac{\mathrm{d}^2\chi_\mathrm{b}}{\mathrm{d}\bar{\varepsilon}\mathrm{d}b}/\pi a_0^2 = \frac{2^6}{3\pi}\frac{\alpha^3}{\bar{E}}\bar{b}\left[\left|\int_0^\infty \mathrm{d}\tau\bar{b}\cos\left(\frac{\bar{\varepsilon}\tau}{2\sqrt{\bar{E}}}\right)G(\bar{b},\tau,\bar{\Lambda},\gamma)\right|^2 + \left|\int_0^\infty \mathrm{d}\tau\,\tau\sin\left(\frac{\bar{\varepsilon}\tau}{2\sqrt{\bar{E}}}\right)G(\bar{b},\tau,\bar{\Lambda},\gamma)\right|^2\right], \quad (12)$$

where the characteristic bremsstrahlung parameter ξ is represented by $\xi(\bar{\varepsilon},\bar{E}) = \bar{\varepsilon}/2\sqrt{\bar{E}}$, $\bar{\varepsilon} \equiv \varepsilon/Z^2Ry$, ε ($= \hbar\omega$) is the radiation photon energy, and the integrand function $G(\bar{b},\tau,\bar{\Lambda},\gamma)$ is written as

$$G(\bar{b},\tau,\bar{\Lambda},\gamma) = \frac{1}{1+c(\gamma)}\frac{1}{\bar{\Lambda}^2(\bar{b}^2+\tau^2)^2}\Bigg\{\Bigg[2\bar{\Lambda}^3 - \left(\frac{(\bar{b}^2+\tau^2)^{3/2}}{2} + (\bar{b}^2+\tau^2)\bar{\Lambda} + 2(\bar{b}^2+\tau^2)^{1/2}\bar{\Lambda}^2 + 2\bar{\Lambda}^3\right)\mathrm{e}^{-(\bar{b}^2+r^2)^{1/2}/\bar{\Lambda}}\Bigg] + \frac{\gamma}{10}\sum_{p=0}^{1}\sum_{q=0}^{1}(-1)^{p+q+1} \cdot\Bigg[\frac{2\bar{\Lambda}^3}{\left(p\sqrt{\gamma}+2q+1\right)^3} - \left(\frac{(\bar{b}^2+\tau^2)^{3/2}}{2} + \frac{(\bar{b}^2+\tau^2)\bar{\Lambda}}{p\sqrt{\gamma}+2q+1} + \frac{2(\bar{b}^2+\tau^2)^{1/2}\bar{\Lambda}^2}{\left(p\sqrt{\gamma}+2q+1\right)^2} + \frac{2\bar{\Lambda}^3}{\left(p\sqrt{\gamma}+2q+1\right)^3}\right) \cdot\mathrm{e}^{-\left(p\sqrt{\gamma}+2q+1\right)(\bar{b}^2+\tau^2)^{1/2}/\bar{\Lambda}}\Bigg]\Bigg\}. \quad (13)$$

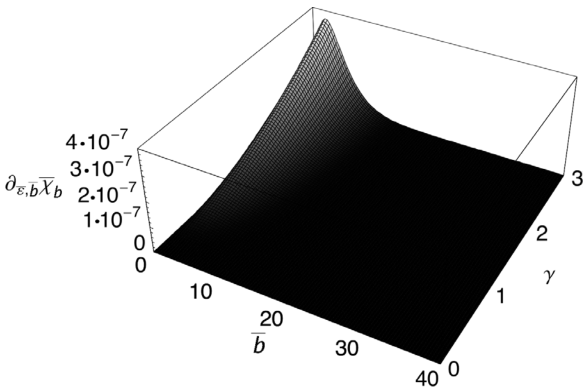

Fig. 1. The surface plot of the scaled differential transition bremsstrahlung radiation cross-section ($\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$) as a function of the scaled impact parameter ($\bar{b}$) and nonideality plasma parameter (γ) for $\bar{\varepsilon} = 0.4$, $\bar{E} = 0.8$, and $\bar{\Lambda} = 50$.

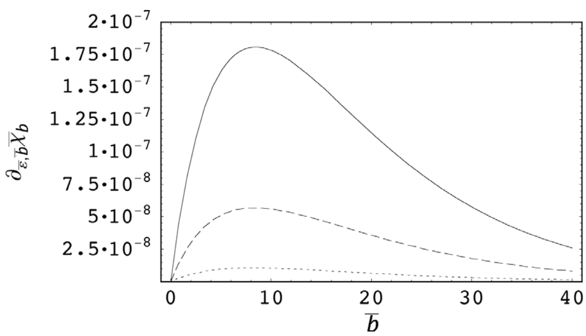

Fig. 2. The soft-photon transition bremsstrahlung radiation cross-section $\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$ as a function of the scaled impact parameter ($\bar{b}$) for $\bar{\varepsilon} = 0.08$, $\bar{E} = 0.8$, and $\bar{\Lambda} = 50$. The solid line is the case of $\gamma = 1$. The dashed line is the case of $\gamma = 0.5$. The dotted line is the case of $\gamma = 0.2$.

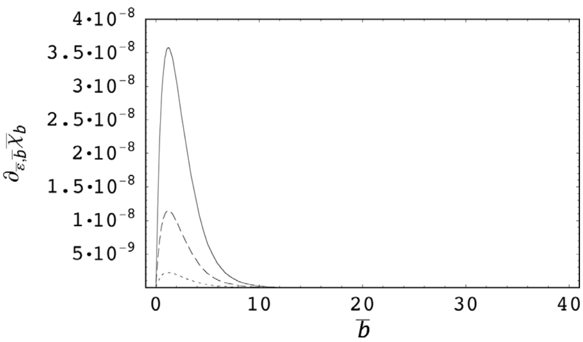

Fig. 3. The hard-photon transition bremsstrahlung radiation cross-section $\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$ as a function of the scaled impact parameter ($\bar{b}$) for $\bar{\varepsilon} = 0.72$, $\bar{E} = 0.8$, and $\bar{\Lambda} = 50$. The solid line is the case of $\gamma = 1$. The dashed line is the case of $\gamma = 0.5$. The dotted line is the case of $\gamma = 0.2$.

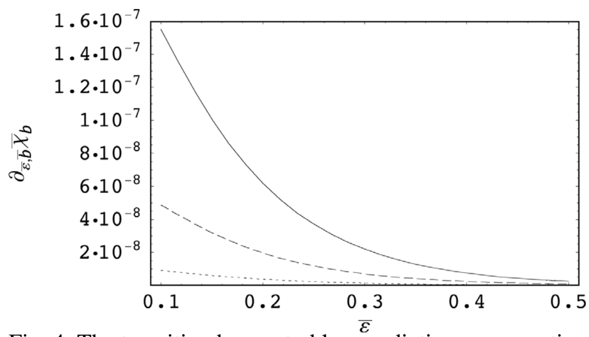

Fig. 4. The transition bremsstrahlung radiation cross-section ($\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$) as a function of the scaled radiation photon energy ($\bar{\varepsilon}$) for $\bar{b} = 10$, $\bar{E} = 0.8$, and $\bar{\Lambda} = 50$. The solid line is the case of $\gamma = 1$. The dashed line is the case of $\gamma = 0.5$. The dotted line is the case of $\gamma = 0.2$.

In order to explicitly investigate the nonideal collective effects on the transition bremsstrahlung process in nonideal plasmas, we consider $\bar{E} < 1$ since the polarization effects are expected to be significant for low-energy projectiles and the classical expression of the bremsstrahlung cross-section is known to be also reliable for $v_0 < Z\alpha c$ [12]. Figure 1 represents the surface plot of the scaled differential transition bremsstrahlung radiation cross-section $\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$ [$\equiv (\mathrm{d}^2\chi_{\mathrm{b}}/\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b})/\pi a_0^2$] as a function of the scaled impact parameter $\bar{b}$ and nonideality plasma parameter γ. As shown, it is found that the collective effects significantly enhance the bremsstrahlung radiation cross-section, especially for small impact parameter domains. Figure 2 shows the soft-photon transition bremsstrahlung radiation cross-section $\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$ as a function of the scaled impact parameter $\bar{b}$ for various values of the nonideality plasma parameter γ. It is shown that the domain of the bremsstrahlung emission is wide in the case of soft-photon emissions, and the collective effect is also found to be the most significant near the maximum position of the bremsstrahlung cross-section. Figure 3 shows the hard-photon transition bremsstrahlung radiation cross-section $\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$ as a function of the scaled impact parameter $\bar{b}$ for various values of the nonideality plasma parameter γ. From this figure, it is shown that the domain of the bremsstrahlung emission is significantly reduced with an increase of the radiation photon energy. It is also found that the bremsstrahlung radiation cross-section decreases with increasing radiation photon energy. In addition, Fig. 4 presents the transition bremsstrahlung radiation cross-section $\partial_{\bar{\varepsilon},\bar{b}}\bar{\chi}_{\mathrm{b}}$ as a function of the scaled radiation photon energy $\bar{\varepsilon}$ for various values of the nonideality plasma parameter γ. As it is seen, the collective effects decrease with an increase of the radiation photon energy.

Thus, we understand that the bremsstrahlung spectra in soft-photon regions can be used to explore the nonideal collective phenomena in nonideal plasmas. From these results, we have shown that the collective effects play very important roles in the transition bremsstrahlung process due to the polarization interaction between the electron and Debye shielding cloud in nonideal plasmas. These results would provide useful information on the continuum emission spectrum due to the transition bremsstrahlung process in plasmas.

Acknowledgements

The authors are grateful for useful discussions with Professor W. Hong. This work was supported by the Catholic University of Daegu. The authors would like to thank the anonymous referees for suggesting improvements to this text.

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